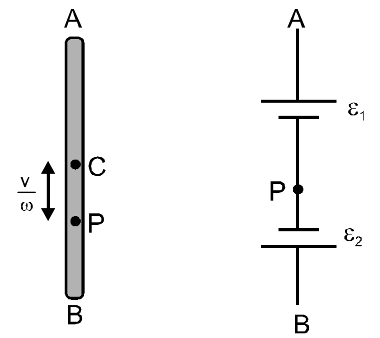
DPP No. # 05

1. Point P is at instantaneous rest,



$$\varepsilon_1 = |v_P - v_A| = \frac{1}{2}B\omega\left(\frac{\ell}{2}+\frac{v}{\omega}\right)^2$$

$$\varepsilon_2 = |v_P - v_B| = \frac{1}{2}B\omega\left(\frac{\ell}{2}-\frac{v}{\omega}\right)^2$$

$$|v_A - v_B| = \varepsilon_1 - \varepsilon_2$$

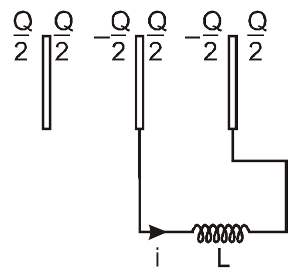
$$|v_A - v_B| = B\ell v$$

2. $\frac{X_L}{R} = \sqrt{3} \Rightarrow X_L = R\sqrt{3}$

$$i = \frac{100}{\sqrt{(10\sqrt{3})^2+(10)^2}} = 5A$$

$$L = \frac{10\sqrt{3}}{100\pi} = \frac{\sqrt{3}}{10\pi}\,H$$

3.

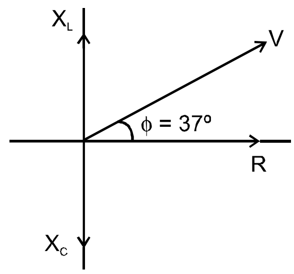


$$\frac{\left(\frac{Q}{2}\right)^2}{2C} = \frac{1}{2}Li_0^2$$

$$\Rightarrow \quad I_0 = \frac{Q}{2\sqrt{LC}}$$

4. impedance $z = \sqrt{(8-2)^2+(8)^2} = 10\ \Omega$

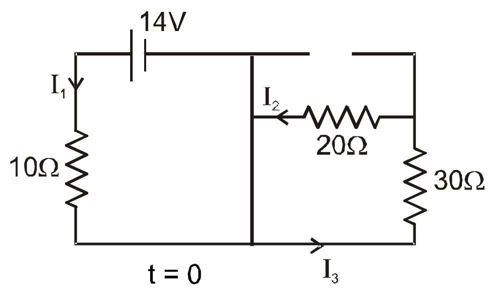
current lags voltage by 37º, then



$$i = \frac{10}{10}\sin(50\pi t - 37º)$$

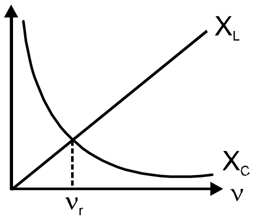
$$V_{AB} = i \times R = 8\sin(50\pi t - 37º)$$

5.



$$I_1 = \frac{14}{10} = 1.4$$

$$I_2 = I_3 = 0$$

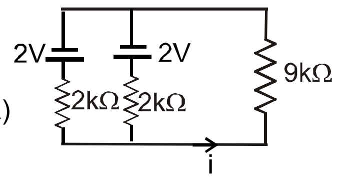
6. 

From graph, When frequency is increased more then resonating frequency (X_C ~ X_L) will increase hence impedence of the circuit will increase

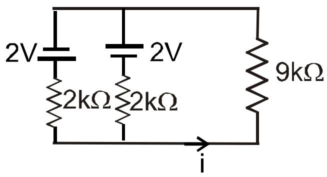
7. Only for resonating fraquency circuit is able to drive appreciable current. So we can use these type of cicuit in tuining of radio and TV for selecting perticular frequency sent by a perticular sorce.

8. $V_L = 8V$, $V_R = 6V$, $V = \sqrt{V_L^2 + V_R^2} = 10\ V$

power factor = $\cos\phi = \dfrac{V_R}{V} = \dfrac{6}{10} = 0.6$

9. (A) 

$i = \dfrac{2}{9+1} = 0.2\ A$

(B) 

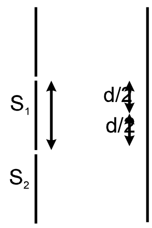
$i = 0$

10. Fringe width = $n\dfrac{\lambda D}{d}$

From given situation

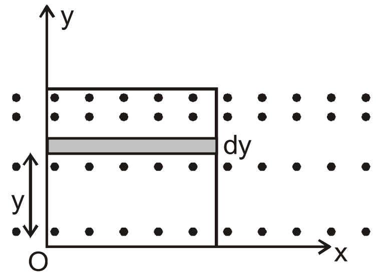
F.W. = $\dfrac{d}{2}$

$\Rightarrow \quad n\dfrac{\lambda D}{d} = \dfrac{d}{2} \quad \Rightarrow \quad \lambda = \dfrac{d^2}{2nD}$



Hence (C) is possible.

11.



$\phi = \int \vec{B}.\overrightarrow{dA}$

$\phi = \int cy^3t^2a\,dy$

$\phi = ct^2a.\int_0^a y^3dy$

$\phi = ct^2a.\frac{a^4}{4}$

$\phi = \frac{ct^2a^5}{4}$

$e = \left|-\frac{d\varphi}{dt}\right|$

Now induced e.m.f.

$e = \left|-\frac{2cta^5}{4}\right| = \frac{2cta^5}{4} = \frac{cta^5}{2}$

12. For 100th maximum

d sin θ = 100 λ

$\sin\theta = \frac{100 \times 5000 \times 10^{-9}}{1 \times 10^{-3}} = \frac{5 \times 10^{-4}}{10^{-3}} = 0.5 = \frac{1}{2}$

$\therefore$ y = D tan θ

= 1 × tan 30

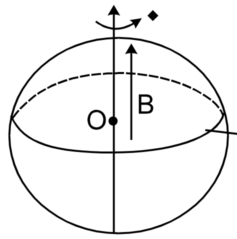
$= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1.732}{3} = 0.577$

13. The area vector of loop $\vec{A} = \pm \ell b\,\hat{k}$

& $\vec{B} = 20t\,\hat{i} + 10\,t^2\,\hat{j} + 50\hat{k}$

$\therefore$ Magnetic flux is $\phi = \vec{B}\,.\,\vec{A} = \pm 50\,\ell b$

$\therefore$ $emf = \frac{d\phi}{dt} = 0$

14. 

the equator can be seen as a conducting ring of radius R_e revolving with angular velocity ω in a perpendicular magnetic field B.

$\therefore$ Potential difference. across its center and periphery = $\frac{B\omega R_e^2}{2}$

Potential at pole = potential of the axis of earth i.e. potential at point O

$\therefore \quad V_{equator} - V_{pole} = \frac{B\omega {R_e}^2}{2}$.

15. $\frac{dq}{dt} = i = 0 \qquad Q \rightarrow max$

$E_C = \frac{Q^2}{2c} \Rightarrow$ max

$E_L = \frac{Li^2}{2} \Rightarrow$ zero

16. Let at time t the angle between magnetic field and area vector(semicircle) be θ, then $\theta = wt$

$$\phi = \vec{B}\cdot\vec{S} = \frac{\pi a^2 B}{2}\cos\omega t.$$

$$\varepsilon = -\frac{d\phi}{dt} = \frac{\pi B a^2 \omega}{2}\sin\omega t$$

$$\varepsilon_0 = \frac{\pi B a^2}{2\sqrt{LC}} \text{ peak emf}$$

Since the circuit is in resoanance,

$$|z| = R \qquad \Rightarrow \qquad i_0 = \frac{\pi B a^2}{2R\sqrt{LC}} \text{ peak current}$$

$$i_{rms} = \frac{i_0}{\sqrt{2}} \qquad \Rightarrow \qquad i_{rms} = \frac{\pi B a^2}{2R\sqrt{2LC}}$$

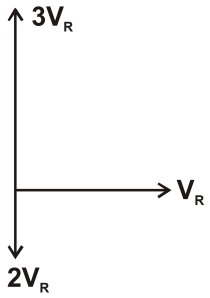
$$U_C = \frac{1}{2}CV_0^2 \rightarrow \text{max. energy}, \ V_0 \rightarrow \text{peak voltage}$$

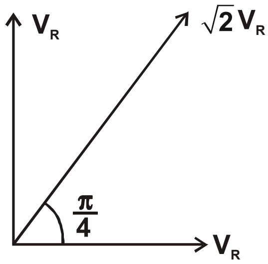
$$V_0 = i_0 X_c = \frac{i_0}{C\omega} = \frac{i_0\sqrt{LC}}{C}$$

$$U_C = \frac{1}{2}C \times \frac{\pi^2 B^2 a^4}{4R^2C^2} = \frac{\pi^2 B^2 a^4}{8R^2 C}$$

$$P_{Ext.} = P_{Dissipated} = \varepsilon_0 i_0 = \frac{\pi B a^2}{2\sqrt{LC}} \times \frac{\pi B a^2}{2R\sqrt{LC}}, \qquad P_{Ext.} = \frac{\pi^2 B^2 a^4}{4LCR}$$

17. V_R, V_L, V_C are r.m.s voltage across the R, L & C respectively





$\theta = \frac{\pi}{4}$

P.F. = $100\theta = \frac{1}{\sqrt{2}}$

$\sqrt{2}\,V_R = 220$

$V_R = \frac{220}{\sqrt{2}}$ = 156 V

18. Let N be the number of fringes within the length x, then we have,

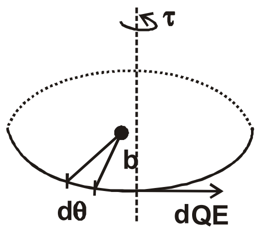
$\beta N = x \Rightarrow \frac{D\lambda}{d} N = x \Rightarrow N = \frac{xd}{\lambda D}$

At any time t

$N = \frac{x}{\lambda D}(d + vt)$

$\frac{dN}{dt} = \frac{xv}{\lambda D}$.

19. Changing magnetic field (at switching off B_0 to zero) induce electric field in such a way to restore the upward flux, hence anticlockwise (E) as seen from above.



$\int \vec{E}.\vec{dl} = -\frac{d\phi}{dt} = -\pi a^2 \frac{dB}{dt} = \int E dl$

There is force on small element dQ of ring, tangentially

Now this force produces torque about axis of ring to rotate in anticlockwise sense, so,

$\tau = \int dQE \times b = \int \lambda d\ell\, Eb = \lambda b \int E\, d\ell = \lambda b \pi a^2 \frac{dB}{dt}$

so Impulse of torque

$\int \tau\, dt = \lambda b \pi a^2 \int_{B_o}^{o} dB = \int \tau dt = \lambda b \pi a^2 B_o$

$L_f - L_i = \Delta L = \int \tau dt = \lambda b \pi a^2 B_o = I\omega$ (in magnitude)

It is independent of time taken

$I\omega_f - I\omega_i = \lambda b \pi a^2 B_0$

Where I is moment of inertia

So , $\omega_f = \frac{\lambda b \pi a^2 B_o}{mR^2}$

20. Because both inductors are in parallel

$\therefore\ L_1I_1 = L_2I_2$

$$\frac{U_1}{U_2} = \frac{\frac{1}{2}L_1 I_1 I_1}{\frac{1}{2}L_2 I_2 I_2} = \frac{I_1}{I_2} = \frac{L_2}{L_1}$$

21. The parth difference

$\Delta x = (\mu_A - 1)t_A - (\mu_B - 1)t_B$

$\Rightarrow \Delta x = \mu_A t_A - t_A - \mu_B t_B + t_B$

$\Rightarrow \Delta x = t_B - t_A$

if $t_B = t_A$ $\Rightarrow \Delta x = 0$

$\Rightarrow$ no shift

if $t_B > t_A$ or $t_B < t_A$

$\Delta x \neq 0$

$\Rightarrow$ central maxima may shift towards A or B.

22. For $S_1S_2 = 2.5\lambda$, max path different = 2.5λ

min path different = 0

Between 2.5λ and 0 lie 2λ and $\lambda \Rightarrow$ two circular bright fringes

$n_1 = 2$

For $S_1S_2 = 5.7\lambda$, max. path different = 5.7λ

min path different = 0

Between 5.7λ and 0 lie 5λ, 4λ, 3λ, 2λ, $\lambda \Rightarrow$ Five circular bright fringes.

$\Rightarrow\ n_2 = 5$

$\therefore\ n_2 - n_1 = 5 - 2 = 3$

23. $\varepsilon = -\dfrac{\Delta\phi}{\Delta t} = -\dfrac{(\phi_2 - \phi_1)}{\Delta t}$

$= \dfrac{\phi_1 - \phi_2}{\Delta t}$

$= \dfrac{BA - 0}{\Delta t} = \dfrac{0.5 \times \pi \left(1 \times 10^{-2}\right)^2}{0.5} = \pi \times 10^{-4}$ V

24. Just after the switch is closed, there is no current through the coil and capacitor offers no resistance.

Net Resistance = $\dfrac{9}{2}$ = 4.5 Ω $\Rightarrow$ $i_0 = \dfrac{18}{4.5}$ = 4 A.

25. For R_1 – L branch

$X_L = \omega L = 100 \times \dfrac{\sqrt{3}}{10} = 10\sqrt{3}\ \Omega$, $R_1 = 10\Omega$

$\therefore$ $\tan\phi = \dfrac{X_L}{R_1} = \sqrt{3}$ or $\phi = 60°$

Hence current I_1 lags voltage by 60°.

For R_2 – C branch

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{100 \times \frac{\sqrt{3}}{2} \times 10^{-3}} = \dfrac{20}{\sqrt{3}}\ \Omega$

$\therefore$ $\tan\phi = \dfrac{X_C}{R_2} = \dfrac{1}{\sqrt{3}}$ or $\phi = 30°$

Hence current I_2 leads voltage by 30º.

$\therefore$ The phase difference between I_1 and I_2 is 90º.

The maximum current through $R_1 - L$ branch is

$$= \frac{V_0}{\sqrt{R_1^2 + \omega^2 L^2}} = \frac{200\sqrt{2}}{\sqrt{10^2 + (10\sqrt{3})^2}} = 10\sqrt{2} \text{ amp.}$$

Hence when current through $R_1 - L$ branch is $10\sqrt{2}$ amp., the current through $R_2 - C$ branch will be zero.

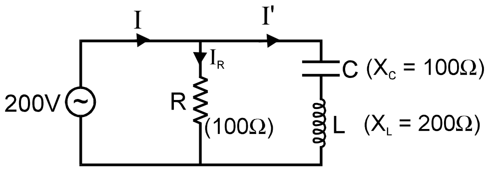
26. $P = V^2/R$

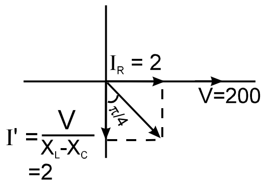
20V peak ac is equivalent to $20/\sqrt{2}$ dc

i.e. 14.14V dc power

$$\frac{\text{dc power}}{\text{ac power}}$$

$$= \frac{\left(20^2/R\right)}{\left[\left(20/\sqrt{2}\right)^2 / R\right]} = \frac{20^2}{\left(20/\sqrt{2}\right)^2} = 2$$

27. 



$$I_R = \frac{V}{R} = \frac{200}{100} = 2A$$

$$I' = \frac{V}{X_L - X_C} = \frac{200}{100} = 2A$$

$$I = \sqrt{I_R^{\,2} + I'^2} = 2\sqrt{2} \text{ Amp.}$$

29. $\frac{\lambda D_1}{d} - \frac{\lambda D_2}{d} = 3 \times 10^{-5}$

$\lambda \times \frac{5 \times 10^{-2}}{10^{-3}} = 3 \times 10^{-5} \Rightarrow$ $\lambda = 0.6 \times 10^{-6} = 6000$ Å.

30. Shift of fringe pattern = $(\mu - 1)\ \frac{tD}{d}$

$\therefore\ \frac{30\text{ D }(4800 \times 10^{-10})}{d} = (0.6)\text{ t }\frac{D}{d}$

$30 \times 4800 \times 10^{-10} = 0.6$

$t = \frac{30 \times 4800 \times 10^{-10}}{0.6} = \frac{1.44 \times 10^{-5}}{0.6} = 24 \times 10^{-6}$

31 to 33

Considering length of line is

$$E = \frac{\sigma}{\varepsilon_0} \Rightarrow V = Eh$$

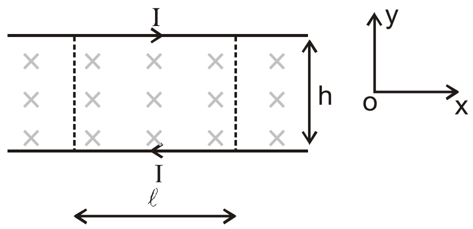
$$V = \frac{\sigma}{\varepsilon_0}h = \frac{Q}{\varepsilon_0 b\ell}.h = c = \frac{Q}{v}$$

$$C = \frac{\varepsilon_0 b\ell}{h}$$

$$\frac{C}{\ell} = \frac{\varepsilon_0 b}{h}$$

$$\Rightarrow B = \frac{\mu_0 K}{2} + \frac{\mu_0 K}{2} = \mu_0 K = \frac{\mu_0 I}{b}$$ (K = current per unit width)

$$\vec{B} = \frac{\mu_0 I}{b}\left(-\hat{k}\right)$$



Consider a rectangular surface as shown in the figure.

Now $\phi = Bh\ell$

$$\phi = \frac{\mu_0 I}{b}.h\ell = LI$$

$$L = \frac{\mu_0 h\ell}{b}$$

$$\frac{L}{\ell} = \frac{\mu_0 h}{b}$$ But

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$c^2 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\mu_0 = \frac{1}{\varepsilon_0 c^2}$$

$$\frac{L}{\ell} = \frac{h}{\varepsilon_0 c^2 b}$$

34 to 36

The fan is operating at 200 V, consuming 1000 W, then $I = \frac{1000}{200} = 5A$

But as coil resistance is 1Ω then power dissipated by internal resistance heat is $P_1 = I^2R = 25W$
If V is net emf across coil then

$$\frac{V^2}{R} = 25 \text{ W} \qquad V = 5 \text{ volt}$$

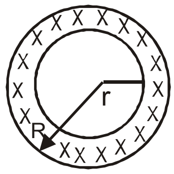
Net emf = source emf – back emf

$V = V_s - e \Rightarrow e = 195$ V

The work done $P_2 = 1000 - 25 = 975$ W.

37. Explanation :

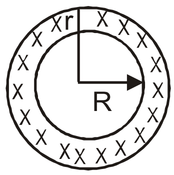
For r < R $\oint E\,dl = A\frac{dB}{dt}$.



$E\,2\pi r = (\pi r^2)\,\alpha$

$E = \frac{r\alpha}{2}$ or $E \propto r$

So, E - r graph is a straight line passing through origin.



At r = R $E = \frac{R\alpha}{2}$

For r > R $E\,2\pi r = (\pi R^2)\,\alpha$

Hence, choice (a) is correct and choices (b), (c) and (d) are wrong.

38. Explanation :

Perpendicular distance between BC and centre O is 10 cm. Component of induced electric field along

the rod $= \frac{d}{2}\frac{dB}{dt}$

Where d = Perpendicular distance from centre to the rod.

Hence, potential difference between the ends of rod

$$v = EI = I.\ \frac{d}{2}\ \frac{dB}{dt}$$

$$= \frac{10}{2} \times 10^{-2} \times 20 \times 10^{-2} \times 2 = 20 \text{ mV}$$

Hence, choice (b) is correct and choices (a), (c) and (d) are wrong.

39. Explanation :

Perpendicular distance between CD and O is 20 cm.

Therefore, induced emf in CD

$$= \frac{d}{2} I \frac{dB}{dt} = \frac{20}{2} \times 10^{-2} \times 20 \times 10^{-2} \times 2$$

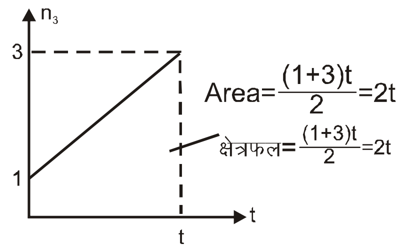
= 40 mV

40. Path difference,

$$\Delta x = n_1\, SS_2 + n_2 S_2 P - \left[(n_1 SS_1 + n_2 S_1 P) - \int_0^t (n_3 - n_2)dx\right]$$

$$= \quad n_1 (SS_2 - SS_1) + n_2(S_2P - S_1P) - \int_0^t n_3 dx + n_2 t$$

In order to get central maxima at centre of screen –



$$O = \frac{2\times(1\times10^{-3})^2}{2\times1} + 0 - 2t + \frac{3t}{2}$$

$0.5\,t = 1\ \mu m.$

$t = 2\ \mu m.$

41. From previous equation :

$$0 = 1\ \mu m + \frac{3yd}{2D} - 0.5\,t$$

$$\frac{3}{2}\frac{yd}{D} = -0.5\ \mu m$$

$$y = -\left(\frac{10^{-6}}{3}\right)\left(\frac{1m}{1\times10^{-3}}\right) = \frac{10^{-3}}{3} = \frac{-1}{3}\text{ mm, below centre.}$$

42. $\beta = \dfrac{\lambda D}{n_2 d} = \dfrac{3000\times10^{-10}\times1\times2}{3\times1\times10^{-3}} = 2\times10^{-4}\text{ m} \quad = 0.2\text{ mm.}$

44. **Explanation :**

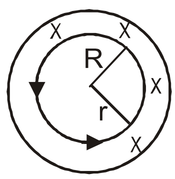
Time varying magnetic field produced electric field known as induced electric field.

So (P) $\rightarrow$ (4)

For r < R

$$\oint E.Dl = -\frac{AdB}{dt}$$

$$E\, 2\pi r = -\pi r^2 \frac{dB}{dt}$$



$$E = -\frac{r}{2}\frac{dB}{dt}$$

So (Q) $\rightarrow$ (2)

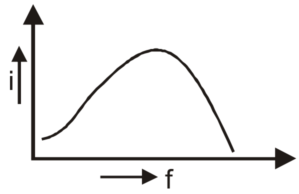
For r > R .

$$E\, 2pr = -\pi R^2 \frac{dB}{dt}\text{ ss,} \quad E = -\frac{R^2}{2r}\frac{dB}{dt}$$

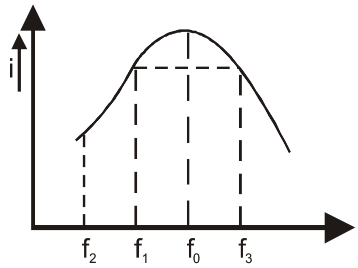
So (R) $\rightarrow$ (3)

If rod is placed along the diameter of magnetic field, then electric field is perpendicular to length of rod.

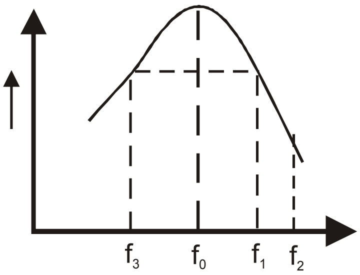
45. power = i^2R graph of 'i' vs f is :



From data, the possibilities are



or



f_0 is resonant frequency $\Rightarrow$ means same as circuit being resistive.
The circuit is capacitative when $f < f_0$ and inductive when $f > f_0$
Power at f_1 and f_3 same $\Rightarrow$ i same $\Rightarrow$ z same

$$\Rightarrow \qquad 2\pi f_1 L - \frac{1}{2\pi f_1 C} = \frac{1}{2\pi f_3 C} - 2\pi f_3 L$$

$$\Rightarrow \qquad 2\pi L\,(f_1 + f_3) = \frac{1}{2\pi C}\left(\frac{1}{f_1} + \frac{1}{f_3}\right)$$

$$\Rightarrow \qquad f_1 f_3 = \frac{1}{4\pi^2 LC} \qquad \Rightarrow \qquad \sqrt{f_1 f_3} = \frac{1}{2\pi\sqrt{LC}} = \frac{\omega_0}{2\pi}$$

$$\text{AM} > \text{GM} \ \Rightarrow \frac{f_1 + f_3}{2} > f_0$$

$$\Rightarrow \qquad \text{Inductive at frequency} = \frac{f_1 + f_3}{2}.$$